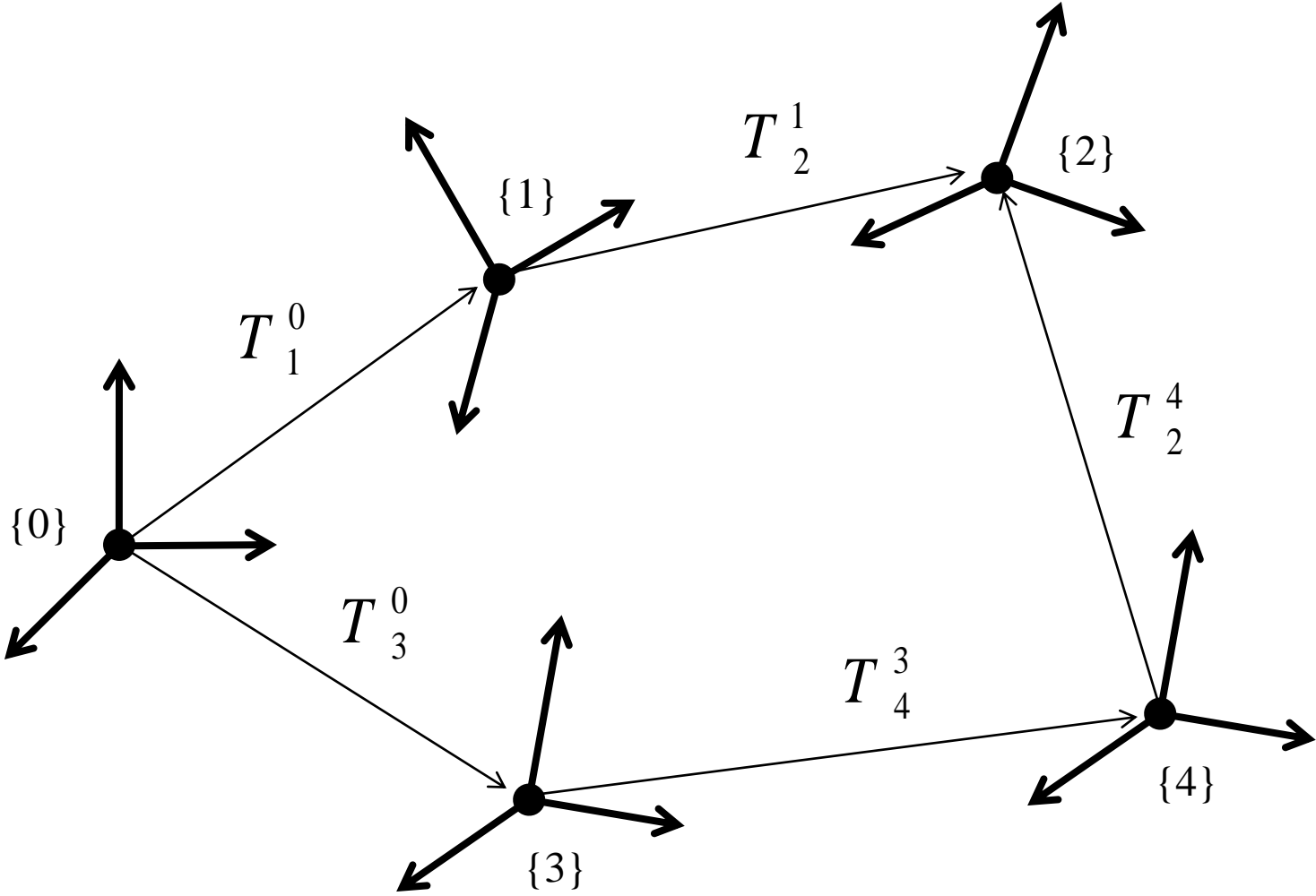


Day 06

Forward Kinematics

# Transform Equations



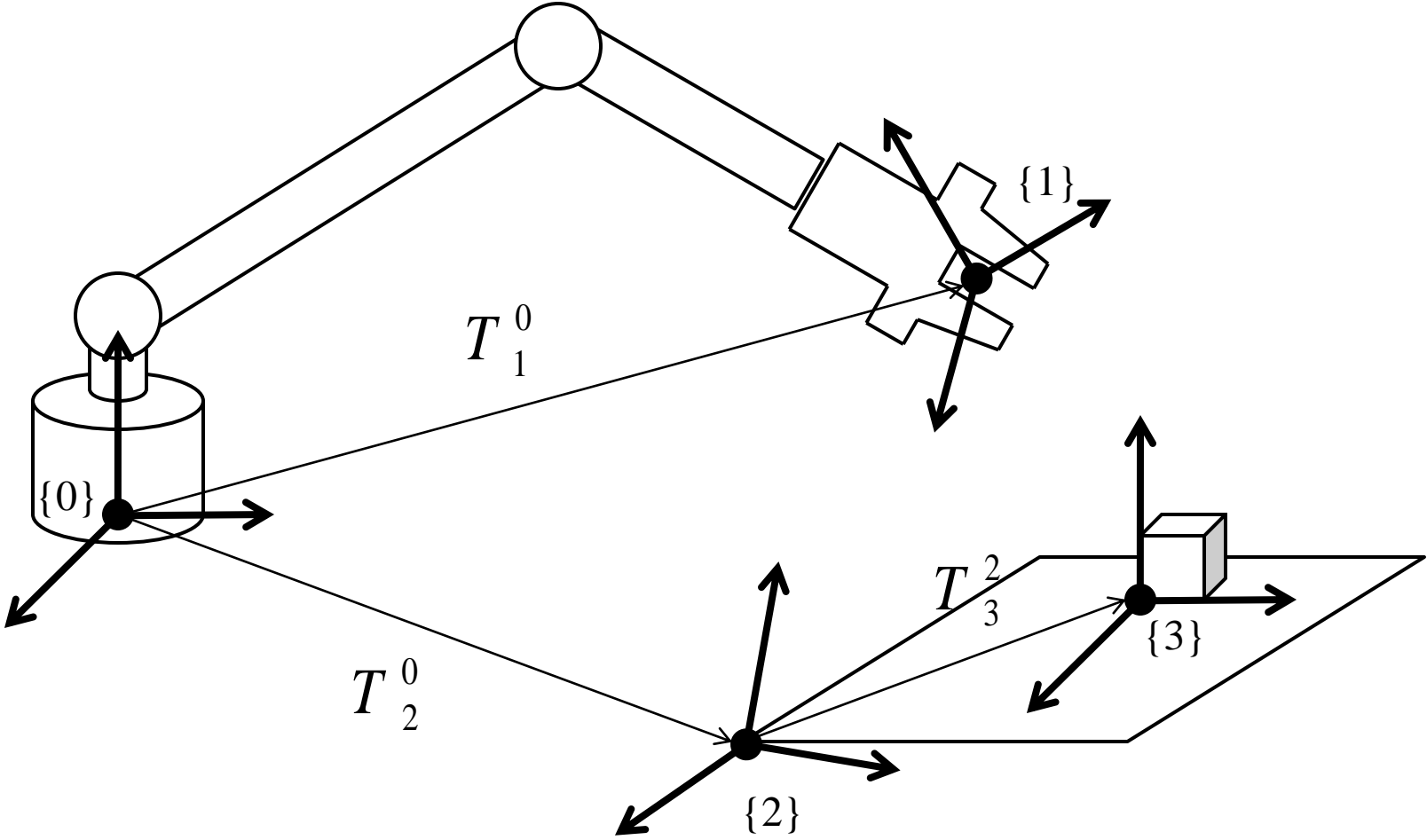
# Transform Equations

- ▶ give expressions for:

$$T_2^0$$

$$T_4^3$$

# Transform Equations



# Transform Equations

► how can you find

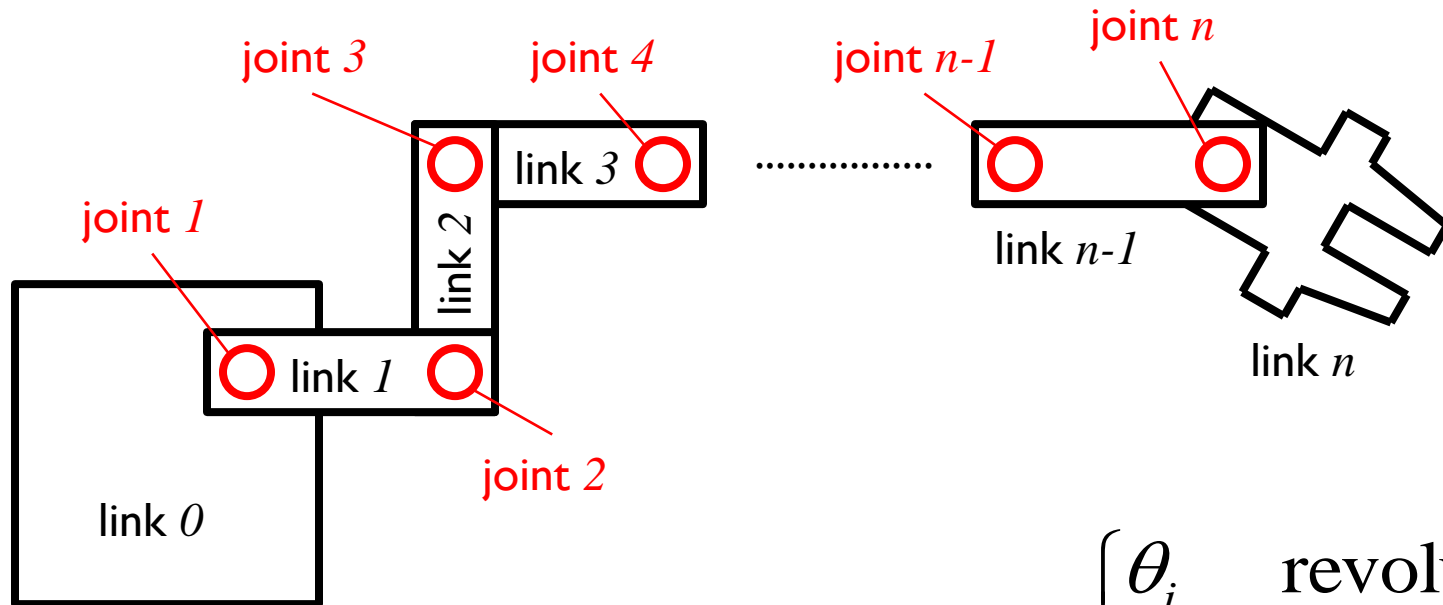
$$T_1^0$$

$$T_2^0$$

$$T_3^2$$

$$T_3^1$$

# Links and Joints

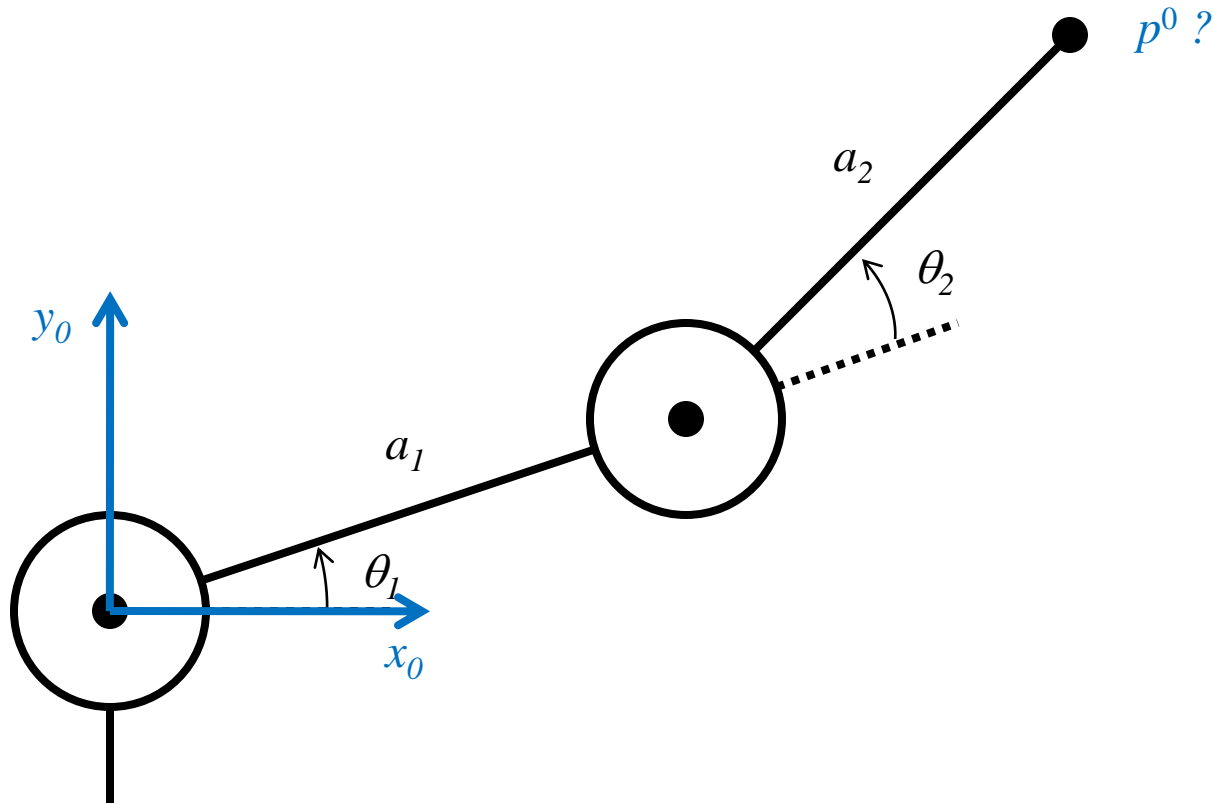


- ▶  $n$  joints,  $n + 1$  links
- ▶ link 0 is fixed (the base)
- ▶ joint  $i$  connects link  $i - 1$  to link  $i$ 
  - ▶ link  $i$  moves when joint  $i$  is actuated

$$q_i = \begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$$

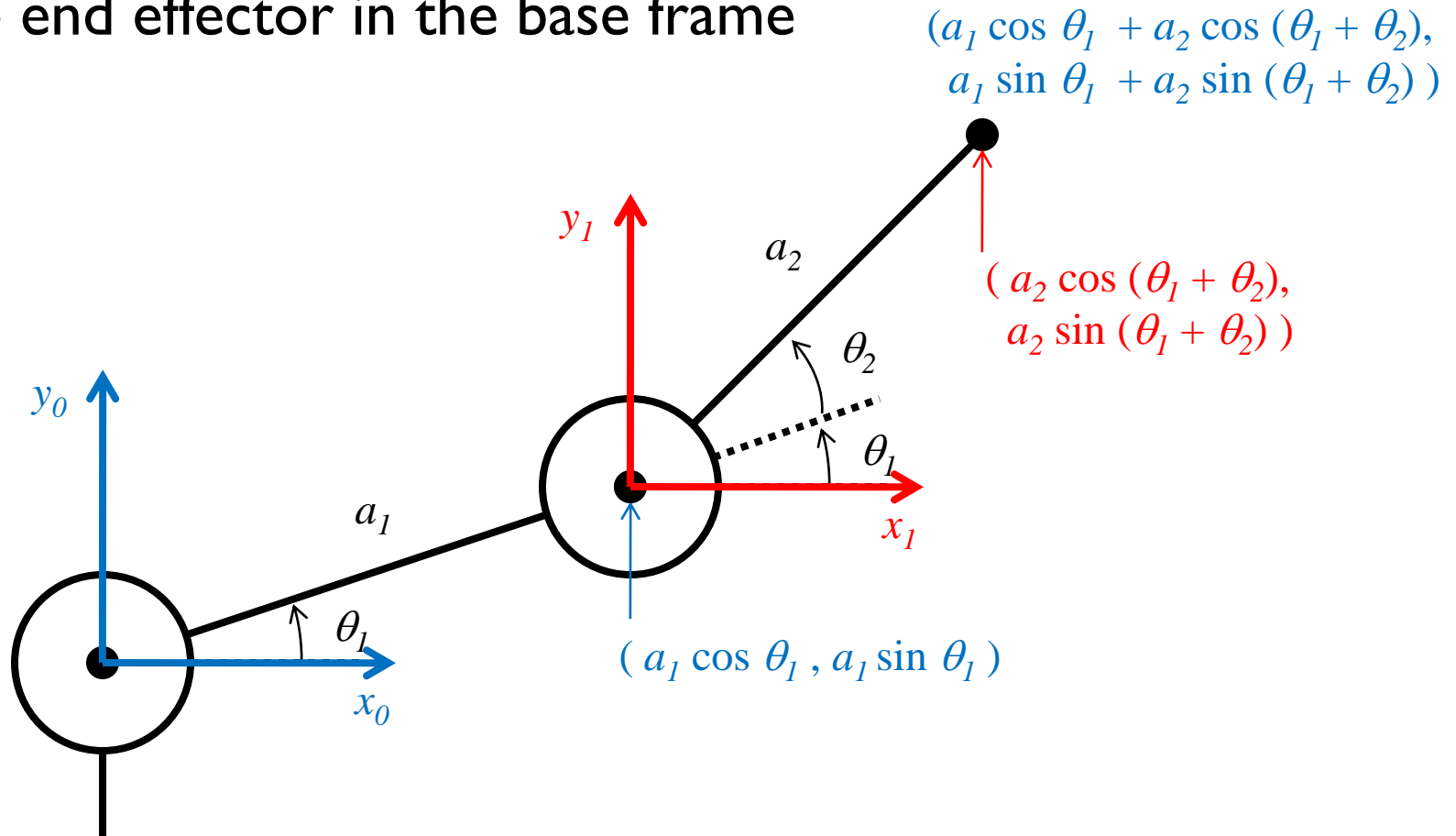
# Forward Kinematics

- ▶ given the joint variables and dimensions of the links what is the position and orientation of the end effector?



# Forward Kinematics

- ▶ because the base frame and frame  $I$  have the same orientation, we can sum the coordinates to find the position of the end effector in the base frame





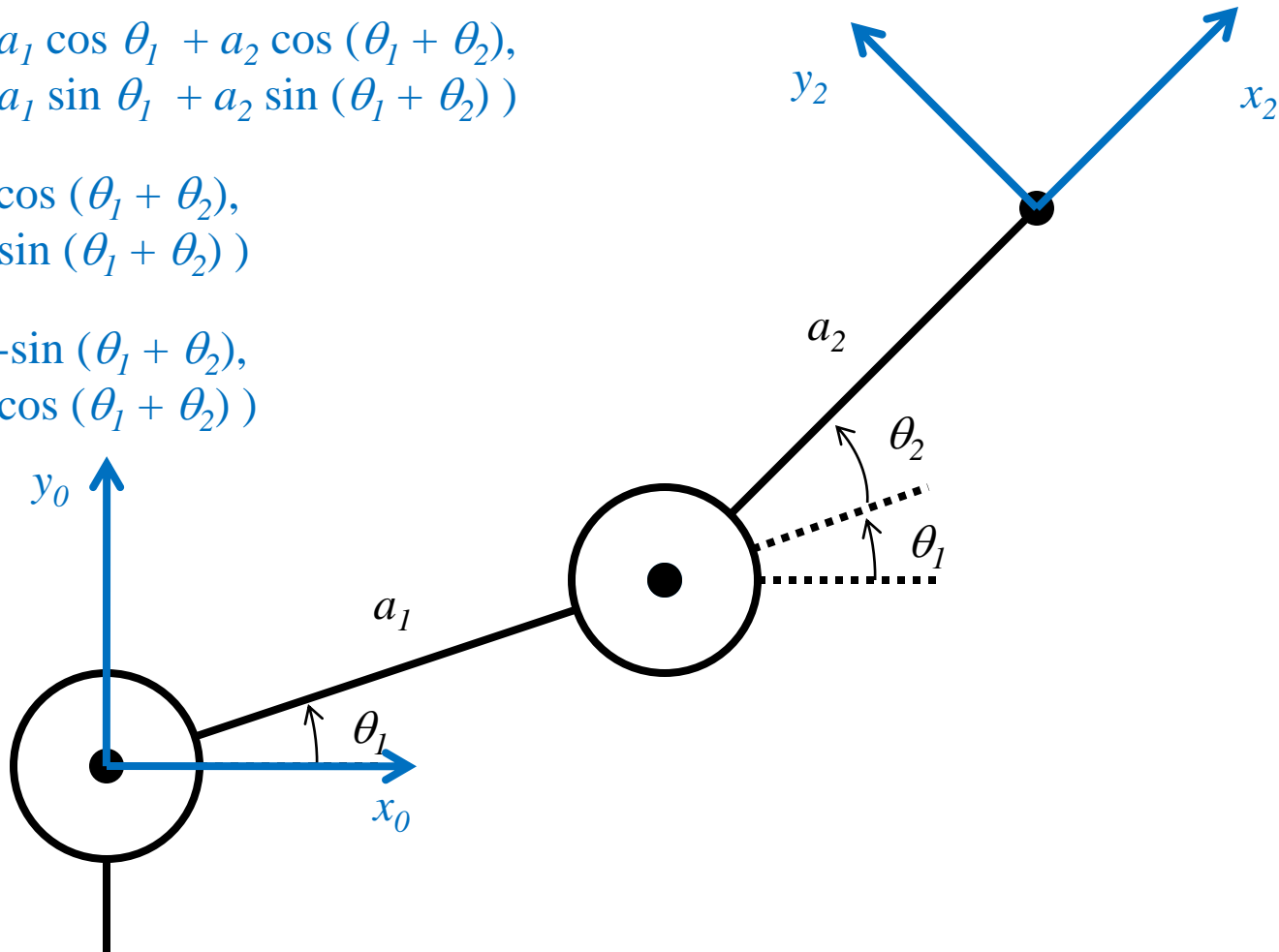
# Forward Kinematics

## ► from Day 02

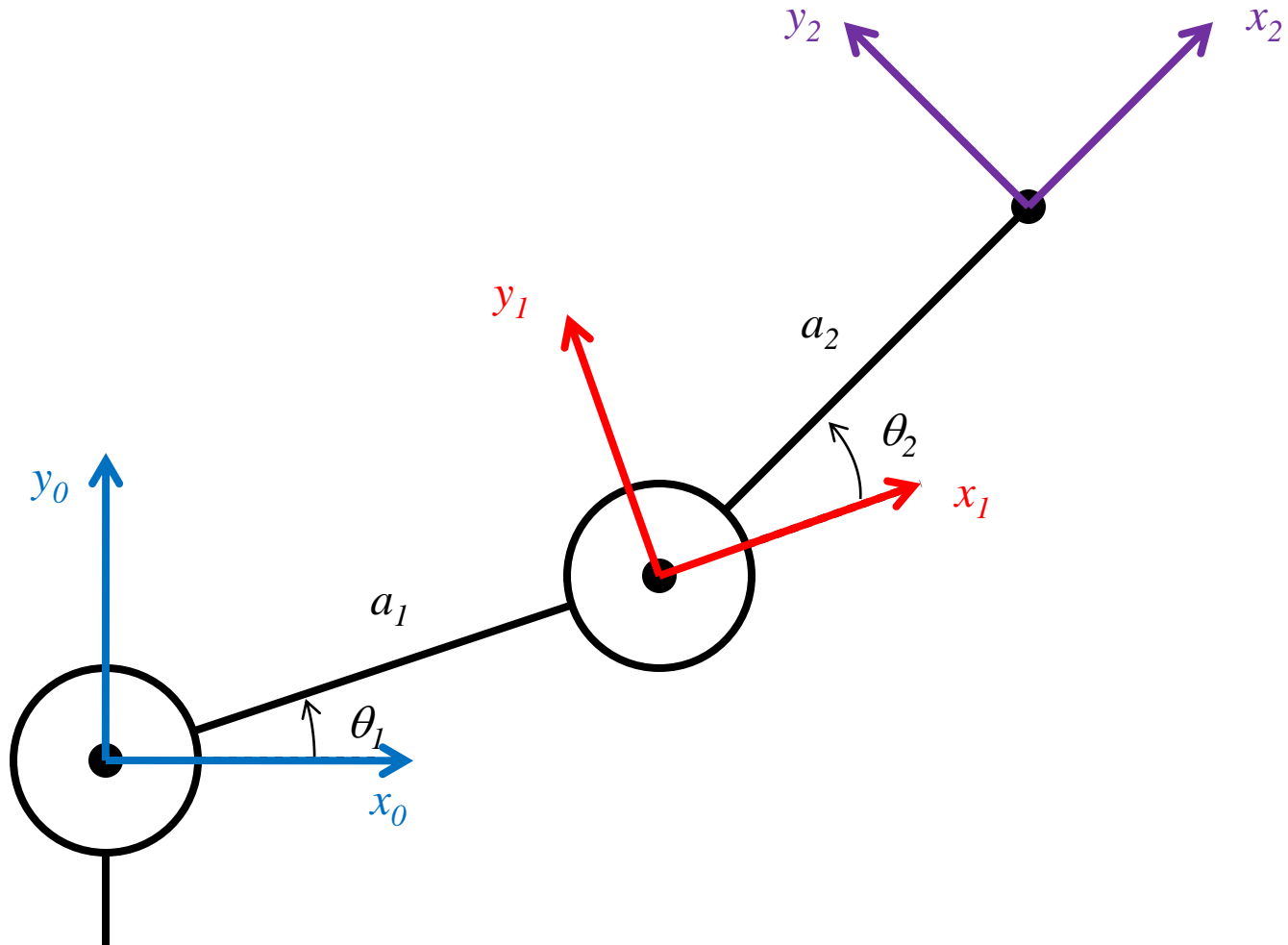
$$p^0 = (a_1 \cos \theta_1 + a_2 \cos (\theta_1 + \theta_2), \\ a_1 \sin \theta_1 + a_2 \sin (\theta_1 + \theta_2))$$

$$x_2 = (\cos (\theta_1 + \theta_2), \\ \sin (\theta_1 + \theta_2))$$

$$y_2 = (-\sin (\theta_1 + \theta_2), \\ \cos (\theta_1 + \theta_2))$$



# Frames



# Forward Kinematics

- ▶ using transformation matrices

$$T_1^0 = R_{z, \theta_1} D_{x, a_1}$$

$$T_2^1 = R_{z, \theta_2} D_{x, a_2}$$

$$T_2^0 = T_1^0 T_2^1$$